ISI – Bangalore Center – B Math - Physics II – Mid Semestral Exam

Date: 10th September 2018. Duration of Exam: 3 hours

Total marks: 60. ANSWER ALL QUESTIONS

Unless otherwise mentioned, all systems below refer to "homogeneous P,V,T" systems. Wherever the material mentioned is a gas, the amount of gas is one mole. Unless explicitly stated, it is not an "ideal" gas.

Q 1. [Total Marks:4+2+4=10]

a.) Derive the following relationship between adiabatic compressibility and isothermal compressibility: $\kappa_{ad} = \frac{c_v}{c_p}\kappa_T$ where $\kappa_T = -\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_T$ $\kappa_{ad} = -\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_{ad}$.

b.) In Figure 1, two curves, one isothermal and the other adiabatic, are plotted in the PV diagram of a gas, not necessarily an ideal gas. The curves are labelled as A and B. Identify which of these curves is adiabatic and which one is isothermal. Justify your answer in light of the result in part a.)

b.) From the result of part a.) above, find the relationship between the pressure and volume of a gas whose equation of state is given by $P(V-b) = RT$ during a quasistatic adiabatic change.

Q2. [Total Marks:6+2+5+2=15]

Consider an engine working in a reversible cycle and using an ideal gas as the working substance with a constant heat capacity C_p.

The cycle consists of two processes at constant pressure, joined by two adiabatics as shown in Figure 2.

a.) Find the efficiency of this engine in terms of P_1 and P_2 and $\gamma = \frac{C_p}{C_v}$.

b.) Which of the temperatures T_a, T_b, T_c, T_d is the highest and which is the lowest?

c.) Suppose a Carnot engine using the same ideal gas works between the highest and lowest temperatures found in part b.) Determine which engine has greater efficiency.

d.) Suppose the Carnot engine in part c.) uses a mixture of non ideal gases. Discuss how that may affect the answer of part c.).

Q3.[Total Marks:2+4+4=10]

a.) State the Clausius theorem for a system exchanging heat with external reservoirs and undergoing a cyclic process.

b.) Use Clausius theorem to show that the entropy of an isolated system undergoing spontaneous changes from an intial state to a final state can not decrease.

c.) Show that the change of entropy of an ideal gas from an initial state (P_i, T_i) to a final state (P_f, T_f) is given by $\Delta S = C_p \ln\left(\frac{T_f}{T_i}\right) - R\ln\left(\frac{P_f}{P_i}\right)$.

Q4.[Total Marks:5+6+4=15].

One mole of an ideal gas with constant heat capacity C_v is kept at atmospheric pressure *P* in a cylindrical vessel of uniform cross-section by using a massless frictionless piston. The gas is thermally insulated and initially occupies a volume *V* and has temperature *T*. Ignore any effect of gravity other than the presence of atmospheric pressure.

The gas undergoes the following transformations. See Figure 3.

Step 1. The piston is suddenly and instantaneously pulled up so that volume of the gas becomes 2V. The piston is held there with a clamp for a brief period so that the gas reaches a state of equilibrium.

Step 2. The clamp of the piston is then released, and the piston allowed to move to reach a state of mechanical equilibrium. Let the final volume and temperature of the gas be V_f and T_f.

a.) Determine the change in pressure, temperature, internal energy and entropy of the gas after step 1. (You can use the entropy of an ideal gas formula without deriving it .)

b.) Determine T_f as a function of T, C_v, R.

c.) Show that the change of entropy from the initial state to the final state at the end of step 2 is given by $C_p \ln(1+\frac{2R}{C_v})$.

Q5. [Total Marks:4+2+4=10]

Using directly the fact that entropy is a state function or by using Maxwell relations, show that

$$\left(\frac{\partial U}{\partial V}\right)_T = T\left(\frac{\partial P}{\partial T}\right)_V - P$$

b.) Show that for any substance with the equation of state having the form $PV = f(T)$, *U* is only an function of *T*.

c.) Suppose a gas has the following properties: its internal energy per unit volume, u , is a function of temperature *T* only and the pressure of the gas P is given by $P = \frac{1}{3}u$. Using the result in part a.)

above find the temperature dependence of u, i.e. find the functional form of $u(T)$ up to a constant.

Results you may use:

1. Quasistatic adiabatic expansion of an idal gas: PV^{c_p/c_v} = constant.

2. Maxwell Relations:

$$\left(\frac{\partial T}{\partial V}\right)_S = -\left(\frac{\partial P}{\partial S}\right)_V$$
$$\left(\frac{\partial T}{\partial P}\right)_S = \left(\frac{\partial V}{\partial S}\right)_P$$
$$\left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial P}{\partial T}\right)_V$$
$$\left(\frac{\partial S}{\partial P}\right)_T = -\left(\frac{\partial V}{\partial T}\right)_P$$